AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 2/11/10 AT 8:00 AM

Name ______________________________
Period _____

SOLUTION TO 2/4/10 POW

1. For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\frac{t}{5}$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time t = 0.

(a) Show that the number of mosquitoes is increasing at time t = 6.

Since R(6)=4.438 > 0, the number of mosquitoes is increasing at t = 6. +1 shows that R(6) >0

(b) At time t = 6, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

$R'(6) = -1.913$. Since R ' (6) < 0, the number of mosquoties is inc at a decreasing rate at t = 6.

+1 considers R ' (6), +1 answer with reason

(c) According to the model, how many mosquitoes will be on the island at time t = 31? Round your answer to the nearest whole number.

$1000 + \int_0^{31} R(t)dt = 964.335$. to the nearest whole number, there are 964 mosquitoes.

+1 integral, +1 answer

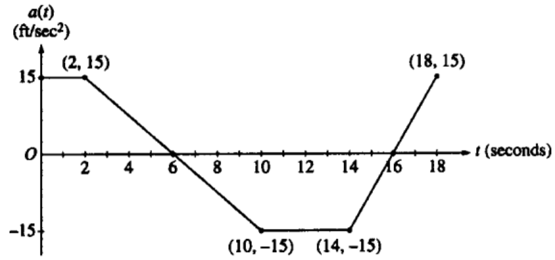
(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

$R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$. $R(t) > 0$ on $0 < t < 2.5\pi$, $R(t) < 0$ on $2.5\pi < t < 7.5\pi$,

$R(t) > 0$ on $7.5\pi < t < 31$. The absolute maximum number of mosquitoes occurs at t = 2.5π or at t = 31.

$1000 + \int_0^{2.5\pi} R(t)dt = 1039.357$. There are 964 mosquitoes at t = 31, so the max number of mosquitoes is 1039, to the nearest whole number. +2 abs max value (+1 integral, +1 answer), +2 analysis (+1 computes interior critical points, +1 completes analysis)

2. A car is traveling on a straight road with velocity 55 ft/sec at time t = 0. For $0 \le t \le 18$ seconds, the car's acceleration a(t) in ft/second squared, is the piecewise linear function defined by the graph below.



a. Is the velocity of the car increasing at t = 2 seconds? Why or why not?

Since v ' (2) = a(2) and a(2) = 15 > 0, the velocity is increasing at t = 2. +1 answer and reason

b. At what time in the interval $0 \le t \le 18$, other than t = 0, is the velocity of the car 55 ft/sec? Why?

At time t = 12 because $v(12) - v(0) = \int_0^{12} a(t)dt = 0$ +1 t = 12, +1 reason

c. On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity in ft/sec, and at what time does it occur? Justify your answer.

The abs max velocity is 115 ft/sec at t = 6. The abs max must occur at t = 6 or at an endpoint.

$v(6) = 55 + \int_0^{6} a(t)dt = 115 > v(0)$. $\int_6^{18} a(t)dt < 0$ so $v(18) < v(6)$ +1 t = 6, +1 abs max velocity, +1 idetifies t = 6 and t = 18 as candidates or indicates that v increases, decreases, then increases, +1 eliminates t = 18

d. At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

The car's velocity is never zero. The abs min occurs at t = 16 where $v(16) = 115 + \int_6^{16} a(t)dt = 115 - 105 = 10 > 0$

+1 answer, +1 reason

3. The number of gallons, P(t), of a pollutant in a lake changes a t the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time t = 0. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

a. Is the amount of pollutant increasing at time t = 9? Why or why not?

$P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$ so the amount is not increasing at this time. +1 answer with reason

b. For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

$P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0, t = (5\ln 3)^2 = 30.174$. $P'(t)$ is negative for $0 < t < (5\ln 3)^2$ and positive for $t > (5\ln 3)^2$. Therefore there is a minimum at t = $(5\ln 3)^2$ +1 sets P'(t) = 0, +1 solves for t, +1 justificaiton

c. Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

$P(30.174) = 50 + \int_0^{30.174} \left(1 - 3e^{-0.2\sqrt{t}}\right)dt = 35.104 < 40$, so the lake is safe.

+1 integrand, +1 limits, +1 conclusion with reason based on integral of P ' (t)

d. An investigator uses the tangent line approximation to P(t) at t = 0 as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

$P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when t = 5. +1 slope of tangent line, +1 answer

POW DUE 2/11/10

The use of a graphing calculator is NOT PERMITTED on these problems. One problem per side of paper.

1. Let f be the function defined by $f(x) = \frac{4x-8}{x^2+5x-14}$

a. Write an equation of the line normal to the graph of f at x = 1.
b. For what values of x is the *derivative* of f, f ' (x), not continuous? Justify your answer.
c. Determine the limit of the derivative at each point of discontinuity found in part (b).
d. Can $\int f(x)dx$ be completed using the method of u-substitution? If yes, complete the integration. If no, explain why u-substitution cannot be used for $\int f(x)dx$.

2. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For 0 < t < 12, the graph of r is concave down. The table above gives selected values of the rate of change $r'(t)$, of the radius of the balloon over the timer interval $0 \le t \le 12$. The radius of the balloon is 30 feet when t = 5. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$).

t (minutes)	0	2	5	7	11	12
r ' (t) (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

a. Estimate the radius of the balloon when t = 5.4 using the tangent line approximate at t = 5. Is your estimate greater than or less than the true value? Give a reason for your answer.
b. Find the rate of change of the volume of the balloon with respect to time when t = 5. Indicate units of measure.
c. Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)dt$ in terms of the radius of the balloon.

d. Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)dt$? Give a reason for your answer.

3. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
v(t) (ft/sec)	-20	-30	-20	-14	-10	0	10
a(t) (ft/sec^2)	1	5	2	1	2	4	2

a. Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|dt$ using a trapezoidal approximation with the three subintervals determined by the table.

b. Using appropriate units, explain the meaning of $\int_0^{30} a(t)dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)dt$.

c. For 0 < t < 60, must there be a time t when v(t) = -5? Justify your answer.

d. For 0 < t < 60, must there be a time t when a(t) = 0? Justify your answer.