AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 2/18/10 AT 8:00 AM

Name ________________________________
Period _____

SOLUTION TO 2/11/10 POW

1. Let f be the function defined by $f(x)=\dfrac{4x-8}{x^2+5x-14}$

a. Write an equation of the line normal to the graph of f at x = 1.

$f(1)=\frac{1}{2}; f(x)=\frac{4(x-2)}{(x+7)(x-2)}=\frac{4}{x+7}\to f'(x)=\frac{-4}{(x+7)^2}\to f'(1)=\frac{-1}{16}\to y-\frac{1}{2}=16(x-1)$ +1 f(1), +1 f ' (x), +1 equation

b. For what values of x is the *derivative* of f, f ' (x), not continuous? Justify your answer.

$f'(x)$ is not continuous at x = -7 or x = 2 since f(x) is not continuous at these points. +1 x = -7, +1 x = 2, +1 reason

c. Determine the limit of the derivative at each point of discontinuity found in part (b).

$\lim_{x\to 2} f'(x)=\frac{-4}{81}; \lim_{x\to -7} f'(x)=-\infty$ +1 $\lim_{x\to 2} f'(x)$, +1 $\lim_{x\to -7} f'(x)=-\infty$

d. Can $\int f(x)dx$ be completed using the method of u-substitution? If yes, complete the integration. If no, explain why u-substitution cannot be used for $\int f(x)dx$.

No. If $u=x^2+5x-14$, then du = 2x + 5. Since the numerator is 4x – 8, integration by u-substitution is not possible.
+1 conclusion with reason

2. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For 0 < t < 12, the graph of r is concave down. The table above gives selected values of the rate of change $r'(t)$, of the radius of the balloon over the timer interval $0 \le t \le 12$. The radius of the balloon is 30 feet when t = 5. (Note: The volume of a sphere of radius r is given by $V=\frac{4}{3}\pi r^3$).

t (minutes)	0	2	5	7	11	12
r ' (t) (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

a. Estimate the radius of the balloon when t = 5.4 using the tangent line approximate at t = 5. Is your estimate greater than or less than the true value? Give a reason for your answer.

$r(5.4)\approx r(5)+r'(5)\Delta t=30+2(0.4)=30.8$ ft Since the graph of r is concave down on the interval 5 < t < 5.4, this estimate is greater than r(5.4). +1 estimate, +1 conclusion with reason

b. Find the rate of change of the volume of the balloon with respect to time when t = 5. Indicate units of measure.

$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}\to \frac{dV}{dt}\Big|_{t=5}=4\pi(30)^2(2)=7200\pi$ ft³ / min +2 $\frac{dV}{dt}$, +1 answer

c. Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)dt$ in terms of the radius of the balloon.

$\int_0^{12} r'(t)dt\approx 2(4.0)+3(2.0)+2(1.2)+4(0.6)+1(0.5)=19.3$ ft. +1 approximation

$\int_0^{12} r'(t)dt$ is the change in the radius, in feet, from t = 0 to t = 12 +1 explanation

d. Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

Since r is concave down, r ' is decreasing on 0 < t < 12. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$

+1 conclusion with reason, +1 units in (b) and (c)

3. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
v(t) (ft/sec)	-20	-30	-20	-14	-10	0	10
a(t) (ft/sec^2)	1	5	2	1	2	4	2

a. Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\,dt$ using a trapezoidal approximation with the three subintervals determined by the table.

$\int_{30}^{60} |v(t)|\,dt$ is the distance in feet that the car travels from t = 30 to t = 60 seconds. +1 explanation

Trapezoidal approximation for $\int_{30}^{60} |v(t)|\,dt$: $A = \frac{1}{2}(14+10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185$ ft +1 value

b. Using appropriate units, explain the meaning of $\int_0^{30} a(t)\,dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)\,dt$.

$\int_0^{30} a(t)\,dt$ is the car's change in velocity in ft/sec from t = 0 to t = 30 sec. +1 explanation

$\int_0^{30} a(t)\,dt = \int_0^{30} v'(t)\,dt = v(30) - v(0) = -14 - (-20) = 6 \text{ ft / sec}$ +1 value

c. For 0 < t < 60, must there be a time t when v(t) = -5? Justify your answer.

Yes. Since v is continuous, v(35) = -10 and v(50) = 0, the IVT guarantees that v(t) = -5 somewhere on (35, 50).
+1 uses IVT, +1 correct conclusion based upon v(35) and v(50).

d. For 0 < t < 60, must there be a time t when a(t) = 0? Justify your answer.

Yes. Since v(0) and v(25) both equal -20, the MVT guarantees that a(t) = 0 somewhere on (0, 25).

+1 v(0) and v(25) both equal -20, +1 uses MVT to justify

+1 units in (a) and (b)

POW DUE 2/18/10

The use of a graphing calculator is REQUIRED on these problems. One problem per side of paper.

1. Let R be the region in the first quadrant which contains the point (0.5, 3) and is bounded by $y = 4 - x^2$ and $y = 1 + 2\sin x$.

a. Find the area of R.

b. Find the volume of the solid generated when R is revolved about the line y = 5.

c. Find the volume of the solid whose base is R and whose cross sections perpendicular to the x-axis are equilateral triangles.

2. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time t = 0, the tank contains 30 gallons of water.

a. How many gallons of water leak out of the tank from time t = 0 to t = 3 minutes?

b. How many gallons of water are in the tank at time t = 3 minutes?

c. Write an expression for A(t), the total number of gallons of water in the tank at time t.

d. At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

3. A particle starts at the point (1, 0) at t = 0 and moves along the x-axis so that at time $t \ge 0$ its velocity v(t) is given by $v(t) = 1 + \frac{t}{1+t^2}$.

a. Determine the maximum velocity of the particle. Justify your answer using the first derivative test.

b. Find an expression for the position s(t) of the particle at time t.

c. What is the limiting value of the velocity as t increases without bound?

d. Determine for which values of t, if any, the particle reaches at the point (101, 0).