AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 2/25/10 AT 8:00 AM

Name ______________________________
Period _____

SOLUTION TO 2/18/10 POW

The use of a graphing calculator is REQUIRED on these problems. One problem per side of paper.

1. Let R be the region in the first quadrant which contains the point (0.5, 3) and is bounded by $y = 4 - x^2$ and $y = 1 + 2\sin x$.

a. Find the area of R.

$\int_0^{1.102} \left[\left(4 - x^2\right) - \left(1 + 2\sin x\right)\right] dx = 1.764$ +1 limits, +1 integrand, +1 answer

b. Find the volume of the solid generated when R is revolved about the line y = 5.

$\pi \int_0^{1.102} \left[\left(5 - \left(1 + 2\sin x\right)\right)^2 - \left(5 - \left(4 - x^2\right)\right)^2\right] dx = 24.943$ +1 limits and constant, +1 integrand, +1 answer

c. Find the volume of the solid whose base is R and whose cross sections perpendicular to the x-axis are equilateral triangles.

$\frac{\sqrt{3}}{4} \int_0^{1.102} \left[\left(4 - x^2\right) - \left(1 + 2\sin x\right)\right]^2 dx = 1.589$ +1 limits and constant, +1 integrand, +1 answer

2. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time t = 0, the tank contains 30 gallons of water.

a. How many gallons of water leak out of the tank from time t = 0 to t = 3 minutes?

$\int_0^3 \sqrt{t+1}\, dt = \frac{2}{3}(t+1)^{3/2} \Big|_0^3 = \frac{14}{3}$ or 4.667, +1 limits, +1 integrand, +1 answer

b. How many gallons of water are in the tank at time t = 3 minutes?

$30 + 8 \times 3 - \frac{14}{3} = \frac{148}{3}$ or 49.333, +1 answer

c. Write an expression for A(t), the total number of gallons of water in the tank at time t.

$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right) dx = 30 + 8t - \int_0^t \left(\sqrt{x+1}\right) dx$ +1 uses initial condition, +1 definite integral

d. At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

$A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$. A'(t) is pos for 0 < t < 63 and negative for 63 < t < 120. Therefore there is a maximum at t = 63. +1 sets A ' (t) = 0, +1 solves for t, +1 justification

3. A particle starts at the point (1, 0) at t = 0 and moves along the x-axis so that at time $t \geq 0$ its velocity v(t) is given by

$$v(t) = 1 + \frac{t}{1+t^2}.$$

a. Determine the maximum velocity of the particle. Justify your answer using the first derivative test.

$v'(t) = \dfrac{(1-t)(1+t)}{(1+t^2)^2} \rightarrow v'(t)$ = 0 at t = 1. Since changes from pos to neg at t = 1, the maximum must occur at t = 0.

$v(0) = 2$ is the maximum velocity +1 v'(t), +1 rejects t = 1, +1 answer

b. Find an expression for the position s(t) of the particle at time t.

$$s(t) = \int \left(1 + \frac{t}{1+t^2}\right) dt$$

$s(t) = t + \frac{1}{2}\ln|1+t^2| + C$ +1 integral of v(t)

$1 = 0 + \frac{1}{2}\ln|1+0^2| + C \rightarrow C = 1$ +1 uses initial condition, +1 constant of integration

$s(t) = t + \frac{1}{2}\ln|1+t^2| + 1$ +1 answer

c. What is the limiting value of the velocity as t increases without bound?

$\lim_{t \to \infty} v(t) = 1$ +1 answer

d. Determine for which values of t, if any, the particle reaches at the point (101, 0).

$101 = t + \frac{1}{2}\ln|1+t^2| + 1$ +1 equation

$t = 95.441$ +1 answer

POW DUE 2/25/10

One question per side of paper.

1. ***The use of a calculator is NOT PERMITTED on this question***. Consider the curve $y = 4x - x^3$ and chord AB joining points A(-3, 15) and B(3, -15) on the curve.

a. Find the x- and y-coordinates of the point(s) on the curve where the tangent line is parallel to chord AB.

b. Write an expression without absolute value for the vertical distance, V, between the curve and the chord AB for $0 < x < 3$.

c. Find the maximum vertical distance between the curve and chord AB for $0 < x < 3$.

2. ***The use of a calculator is REQUIRED on this question***. A particle moves along the x-axis so that at any time t, $0 \le t \le 5$, its velocity is given by $v(t) = \sin t + e^{-t}$. When t = 0, the particle is at the origin.

a. Write an expression for the position x(t) of the particle at any time t, $0 \le t \le 5$

b. Find all values of t for which the particle is at rest.

c. For $0 \le t \le 5$, find the average value of the position function determine din part (a).

d. Find the total distance traveled by the particle from t = 0 to t = 5.

3. ***The use of a calculator is REQUIRED on this question***. Let R be the region enclosed by the graph of $f(x) = \frac{1}{x^2}$, $g(x) = e^{-x}$, and the lines x = 1 and x = k where k > 0.

a. Sketch the graphs of f and g on the interval $[0, k]$.

b. Without using absolute value, set up and evaluate in terms of k, an integral expression that gives A(k), the area of region R.

c. Find $\lim_{k \to \infty} A(k)$

d. Set up, but <u>do not evaluate</u>, an integral expression which gives the volume generated when region R is revolved about the <u>x-axis</u>.