AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 3/4/10 AT 8:00 AM

Name ________________________________
Period _____

SOLUTION TO 2/25/10 POW

1. ***The use of a calculator is NOT PERMITTED on this question***. Consider the curve $y = 4x - x^3$ and chord AB joining points A(-3, 15) and B(3, -15) on the curve.

a. Find the x- and y-coordinates of the point(s) on the curve where the tangent line is parallel to chord AB.

$m = \frac{15 - -15}{-3 - 3} = -5 \to \frac{dy}{dx} = 4 - 3x^2$ +1 slope of AB, +1 $\frac{dy}{dx}$

$\to -5 = 4 - 3x^2$

$\to x = \pm\sqrt{3}$ +1 $x = \pm\sqrt{3}$

$\to \left(\sqrt{3}, \sqrt{3}\right), \left(-\sqrt{3}, -\sqrt{3}\right)$ +1 answer

b. Write an expression without absolute value for the vertical distance, V, between the curve and the chord AB for $0 < x < 3$.

$P = \left(4x - x^3\right) - \left(-5x\right)$ +1 uses y = -5x

$P = 9x - x^3$ +1 answer

c. Find the maximum vertical distance between the curve and chord AB for $0 < x < 3$.

$P' = 9 - 3x^2$ $P'' = -6x$ +1 solves $f'(x) = 0$

$0 = 9 - 3x^2$ $P''\left(\sqrt{3}\right) < 0$ therefore max +1 answer

$x = \pm\sqrt{3}$ $P\left(\sqrt{3}\right) = 6\sqrt{3}$ is the maximum vertical distance +1 justification

2. ***The use of a calculator is REQUIRED on this question***. A particle moves along the x-axis so that at any time t, $0 \le t \le 5$, its velocity is given by $v(t) = \sin t + e^{-t}$. When t = 0, the particle is at the origin.

a. Write an expression for the position x(t) of the particle at any time t, $0 \le t \le 5$

$x(t) = \int \left(\sin t + e^{-t}\right) dt$ +1 integrates v(t)

$x(t) = -\cos t - e^{-t} + C$

$0 = -1 - 1 + C \to C = 2$ +1 solves for C

$x(t) = -\cos t - e^{-t} + 2$ +1 answer

b. Find all values of t for which the particle is at rest.

$0 = \sin t + e^{-t} \to t = 3.183$ +1 answer

c. For $0 \le t \le 5$, find the average value of the position function determine din part (a).

$\frac{1}{5-0}\int_0^5 x(t)\,dt = 1.993$ +1 limits and constant

+1 integrand

+1 answer

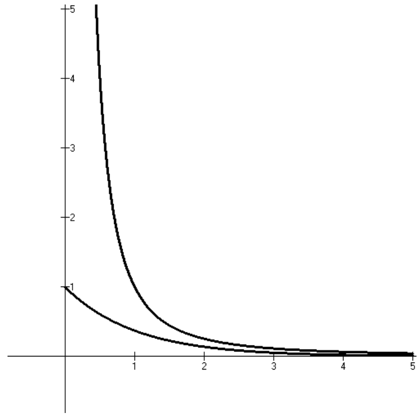
d. Find the total distance traveled by the particle from t = 0 to t = 5.

$\int_0^5 \left|v(t)\,dt\right| = 4.206$ +1 integrates $|v(t)|$

+1 answer

3. ***The use of a calculator is REQUIRED on this question***. Let R be the region enclosed by the graph of $f(x)=\frac{1}{x^2}$, $g(x)=e^{-x}$, and the lines x = 1 and x = k where k > 0.

a. Sketch the graphs of f and g on the interval $[0,k]$. +1 graph



b. Without using absolute value, set up and evaluate in terms of k, an integral expression that gives A(k), the area of region R.

$A(k)=\int_1^k (f(x)-g(x))dx=\left(\frac{-1}{x}+e^{-x}\right)\Big|_1^k$ +1 limits, +1 integrates f(x) - g(x), +1 indefinite integral

$=\left(\frac{-1}{k}+e^{-k}\right)-\left(-1+\frac{1}{e}\right)$ +1 answer

c. Find $\lim_{k\to\infty} A(k)$

$\lim_{k\to\infty} A(k)=1-\frac{1}{e}$ or 0.632 +1 answer

d. Set up, but <u>do not evaluate</u>, an integral expression which gives the volume generated when region R is revolved about the <u>x-axis</u>.

$\pi\int_1^k \left((f(x))^2-(g(x))^2\right)dx$ +1 limits and constant

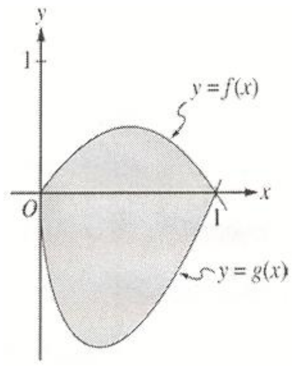
+2 integrand, <-1> each error

max 0/2 if not R^2-r^2

POW DUE 3/04/10

One question per side of paper.
The use of a calculator is REQUIRED on these questions.

1. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure below.



a. Find the area of the shaded region enclosed by the graphs of f and g.

b. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line y = 2.

c. Let h be the function given by h(x) = kx(1 – x) for $0 \leq x \leq 1$. For each k > 0, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

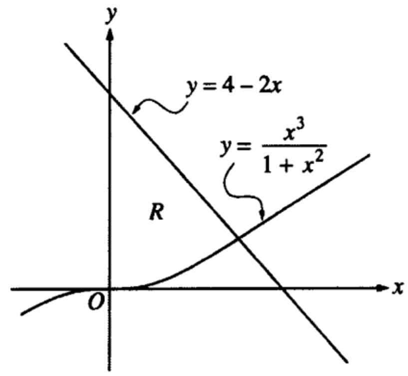
2. Let R be the region enclosed by the graph of $y = \sqrt{x - 1}$, the vertical line x = 10, and the x-axis.

a. Find the area of R.

b. Find the volume of the solid generated when R is revolved about the horizontal line y = 3.

c. Find the volume of the solid generated when R is revolved about the vertical line x = 10.

3. Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1 + x^2}$ and $y = 4 - 2x$, as shown in the figure below.



a. Find the area of R.

b. Find the volume of the solid generated when R is revolved about the x-axis.

c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.