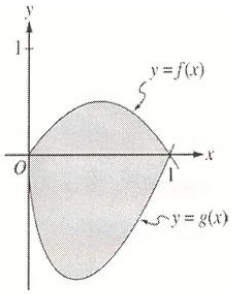
AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 3/11/10 AT 8:00 AM

Name _____
Period _____

SOLUTION TO 3/4/10 POW

1. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure below.



a. Find the area of the shaded region enclosed by the graphs of f and g.

$\text{Area} = \int_0^1 \left(f(x) - g(x)\right)dx = \int_0^1 \left(2x(1-x) - 3(x-1)\sqrt{x}\right)dx = 1.133$ +1 integral, +1 answer

b. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line y = 2.

$\text{Volume} = \pi\int_0^1 \left(\left(2 - g(x)\right)^2 - \left(2 - f(x)\right)^2\right)dx = 16.179$ +1 limits and constant, +2 integrand <-1> each error, +1 answer

c. Let h be the function given by h(x) = kx(1 – x) for $0 \le x \le 1$. For each k > 0, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

$\text{Volume} = \int_0^1 \left(h(x) - g(x)\right)^2 dx = 15$ +2 integrand, +1 answer

2. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line x = 10, and the x-axis.

a. Find the area of R.

$\text{Area} = \int_1^{10} \sqrt{x-1}dx = 18$ +1 limits, +1 integrand, +1 answer

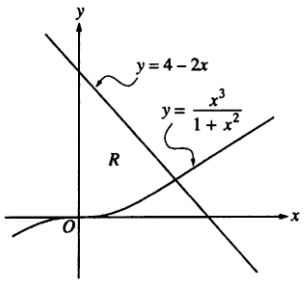
b. Find the volume of the solid generated when R is revolved about the horizontal line y = 3.

$\text{Volume} = \pi\int_1^{10} \left(9 - \left(3 - \sqrt{x-1}\right)^2\right)dx = 212.057$ +1 limits and constant, +1 integrand, +1 answer

c. Find the volume of the solid generated when R is revolved about the vertical line x = 10.

$\text{Volume} = \pi\int_0^3 \left(10 - \left(y^2 + 1\right)\right)^2 dy = 407.150$ +1 limits and constant, +1 integrand, +1 answer

3. Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure below.



a. Find the area of R.

$$\text{Area} = \pi \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2} \right) dx = 3.214$$ +1 integrand, +1 answer

b. Find the volume of the solid generated when R is revolved about the x-axis.

$$\text{Volume} = \pi \int_0^A \left((4-2x)^2 - \left(\frac{x^3}{1+x^2} \right)^2 \right) dx = 31.884 \text{ or } 10.149\pi$$ +2 integrand and constant, +1 answer

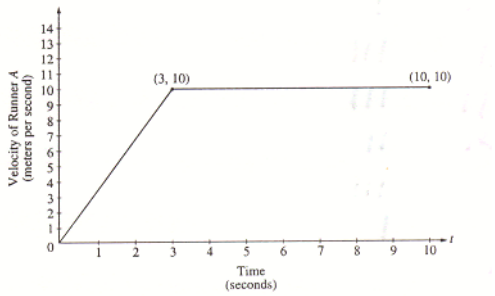
c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$$\text{Volume} = \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2} \right)^2 dx = 8.997$$ +2 integrand <-1> each error, +1 answer

POW DUE 3/11/10

One question per side of paper.
The use of a calculator is REQUIRED on these questions.

1. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$



a. Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure

b. Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.

c. Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

2. A particle moves along the y-axis with velocity given by $v(t) = t\sin\left(t^2\right)$ for $t \ge 0$.

a. In which direction (up or down) is the particle moving at time t = 1.5? Why?

b. Find the acceleration of the particle at time t = 1.5. Is the velocity of the particle increasing at t = 1.5? Why or why not?

c. Given that y(t) is the position of the particle at time t and that y(0) = 3, find y(2).

d. Find the total distance traveled by the particle from t = 0 to t = 2.

3. An object moves along the x-axis with initial position x(0) = 2. The velocity of the object at time $t \ge 0$ is given by $v(t) = \sin\left(\dfrac{\pi}{3}t\right)$

a. What is the acceleration of the object at time t = 4?

b. Consider the following two statements.
Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
Statement II: For $3 < t < 4.5$, the speed of the object is increasing.
Are either or both of these statements correct?
For each statement provide a reason why it is correct or not correct.

c. What is the total distance traveled by the object over the time interval $0 \le t \le 4$?

d. What is the position of the object at time t = 4?