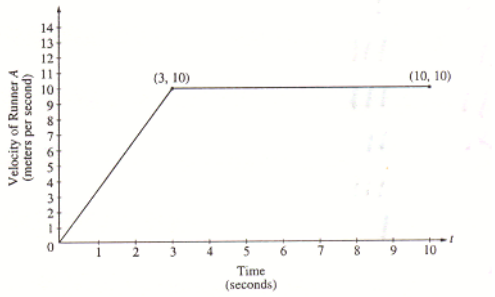
AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 3/18/10 AT 8:00 AM

Name _____
Period _____

SOLUTION TO 3/11/10 POW

1. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$



a. Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure

$\text{Runner A : velocity} = \dfrac{10}{3} \cdot 2 = 6.667 \text{ meters/sec}, \text{Runner B} = v(2) = \dfrac{48}{7} = 6.857 \text{ meters/sec}$

+1 velocity Runner A, +1 velocity Runner B

b. Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.

$\text{Runner A : acceleration} = \dfrac{10}{3} = 3.333 \text{ meters/sec}^2, \text{Runner B} : a(2) = v'(2) = \dfrac{72}{(2t+3)^2}\Big|_{t=2} = \dfrac{72}{49} = 1.469 \text{ meters/sec}^2$

+1 acceleration for Runner A, +1 acceleration for Runner B

c. Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

$\text{Runner A : distance} = \dfrac{1}{2}(3)(10) + 7(10) = 85\text{m}, \text{Runner B : distance} = \displaystyle\int_0^{10} \frac{24t}{2t+3}\,dt = 83.336\text{m}$

+1 method Runner A, +1 answer Runner A, +1 method Runner B, +1 answer Runner B

+1 units in parts a, b, c

2. A particle moves along the y-axis with velocity given by $v(t) = t\sin\left(t^2\right)$ for $t \ge 0$.

a. In which direction (up or down) is the particle moving at time t = 1.5? Why?

$v(1.5) = 1.5\sin(1.5)^2 = 1.167$. Up, because v(1.5) > 0 +1 answer and reason

b. Find the acceleration of the particle at time t = 1.5. Is the velocity of the particle increasing at t = 1.5? Why or why not?

$a(t) = v'(t) = \sin t^2 + 2t^2\cos t^2, a(1.5) = v'(1.5) = -2.048$. No, v is decreasing at 1.5 because $v'(1.5) < 0$

+1 a(1.5), +1 conclusion and reason

c. Given that y(t) is the position of the particle at time t and that y(0) = 3, find y(2).

$y(t) = \int v(t)\,dt = \int t\sin t^2 = \dfrac{-\cos t^2}{2} + C \to y(0) = 3 = -\dfrac{1}{2} + C \to C = \dfrac{7}{2} \to y(t) = \dfrac{-1}{2}\cos t^2 + \dfrac{7}{2} \to y(2) = \dfrac{-1}{2}\cos 4 + \dfrac{7}{2} = 3.826$

$+1\ y(t) = \int v(t)\,dt, +1\ \ y(t) = \dfrac{-1}{2}\cos t^2, +1\, y(2)$

d. Find the total distance traveled by the particle from t = 0 to t = 2.

$\text{distance} = \displaystyle\int_0^2 |v(t)|\,dt = 1.173$ +1 limits, +1 handles change of direction, +1 answer

3. An object moves along the x-axis with initial position x(0) = 2. The velocity of the object at time $t \geq 0$ is given by

$$v(t) = \sin\left(\frac{\pi}{3}t\right)$$

a. What is the acceleration of the object at time t = 4?

$a(4`) = v'(4) = \frac{\pi}{3}\cos\frac{4\pi}{3} = \frac{-\pi}{6}$ or -0.523 +1 answer

b. Consider the following two statements.
Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
Statement II: For $3 < t < 4.5$, the speed of the object is increasing.
Are either or both of these statements correct?
For each statement provide a reason why it is correct or not correct.

On $3 < t < 4.5$, $a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right) < 0$. Statement I is correct since $a(t) < 0$.

Statement II is correct since $v(t) < 0$ and $a(t) < 0$ +1 I correct with reaon +1 II correct, +1 reason for II

c. What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

$\text{Distance} = \int_0^A |v(t)|\,dt = 2.387$ +1 limits, +1 handles change of direction, +1 answer

d. What is the position of the object at time t = 4?

$x(4) = x(0) + \int_0^4 v(t)\,dt = 3.432$ +1 integral, +1 answer

POW DUE 3/18/10

One question per side of paper.
The use of a calculator is REQUIRED on these questions.

1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4\sin\frac{t}{2}$ for $0 \le t \le 30$, where F(t) is measured in cars per minute and t is measured in minutes.

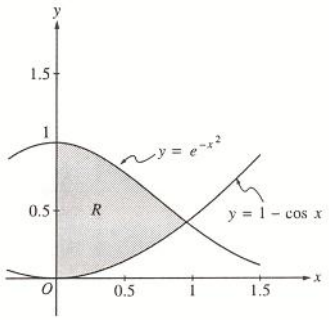
a. To the nearest whole number, how many cars pass through the intersection over the 30-minutes period?

b. Is the traffic flow increasing or decreasing at t = 7? Give a reason for your answer.

c. What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

d. What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

2. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure below.



a. Find the area of the region R.

b. Find the volume of the solid generated when the region R is revolved about the x-axis.

c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

3. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line x = 4.

a. Find the area of the region R.

b. Find the value of h such that the vertical line x = h divides the region R into two regions of equal area.

c. Find the volume of the solid generated when R is revolved about the x-axis.

d. The vertical line x = k divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volume. Find the value of k.