AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 4/15/10 AT 8:00 AM

Name ________________________________
Period _____

SOLUTION TO 4/1/10 POW

The use of a calculator is REQUIRED on these questions. One question per side of paper.

1. A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) for $0 \le t \le 40$are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

a. Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$.

Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

Midpt Sum = $10\left[9.2+7.0+2.4+4.3\right]=229$. The integral gives the total distance in miles that the plane flies during the 40 minutes. +1 $9.2+7.0+2.4+4.3$, +1 answer, +1 meaning with units

b. Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer.

By the Mean Value Theorem, v ‘ (t) = 0 somewhere in the interval (0, 15) and somewhere in the interval (25, 30). Therefore the acceleration will equal 0 for at least two values of t +1 two instances, +1 justification

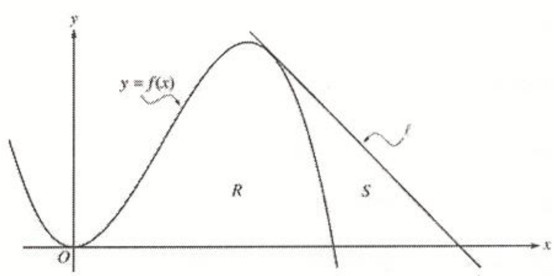
c. The function f, defined by $f(t)=6+\cos\frac{t}{10}+3\sin\frac{7t}{40}$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at t = 23? Indicate units of measure.

$f'(23)=-0.407$ miles per minute^2 +1 answer with units

d. According to the model f, given in part (c) , what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

$ave_{velocity}=\frac{1}{40}\int_0^{40} f(t)\,dt=5.916$ +1 limits, +1 integrand, +1 answer

2. Let f be the function given by $f(x)=4x^2-x^3$, and let ℓ be the line y = 18 – 3x, where ℓ is tangent to he graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the Line ℓ, and the x-axis as shown below.



a. Show that ℓ is tangent to the graph of y = f(x) at the point x = 3.

$f'(x)=8x-3x^2, f(3)=-3, f(3)=9 \rightarrow y-9=-3(x-3)$ which is the equation of ℓ.

+1 f'(3) and f(3), +1 eq of tan line

b. Find the area of S.

f(x) = 0 at x = 4; The line interesects the x-axis at x = 6. Area = $\frac{1}{2}(3)(9)-\int_3^4\left(4x^2-x^3\right)dx=7.916$

+1 limits, +1 integrand, +1 area of triangular region, +1 answer

c. Find the volume of the solid generated when R is revolved about the x-axis.

Volume = $\pi\int_0^4\left(4x^2-x^2\right)^2 dx=156.038\pi$ or 490.208 +1 limits and constant, +1 integrand, +1 answer

3. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\frac{t^2}{2}$. At time t = 0, the particle is at position x = 1.

a. Find the acceleration of the particle at time t = 2. Is the speed of the particle increasing at t = 2? Why or why not?

$a(2) = v'(2) = 1.587$, $v(2) = -3\sin(2) < 0$. Speed is decreasing since a(2) > 0 and v(2) < 0

+1 a(2), +1 speed decreasing with reason

b. Find all times t in the open interval 0 < t < 3 when the particle changes direction. Justify your answer.

$v(t) = 0$ when $\frac{t^2}{2} = \pi \rightarrow t = \sqrt{2\pi}$ or 2.506 Since v(t) < 0 for 0 < t < $\sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi}$< t < 3, the particle changes directions at t = $\sqrt{2\pi}$. +1 t = $\sqrt{2\pi}$ only, +1 justification

c. Find the total distance traveled by the particle from time t = 0 and time t = 3.

$\text{Distance} = \int_0^3 |v(t)\,dt| = 4.333$ +1 limits, +1 integrand, +1 answer

d. During the time interval 0 $\leq$ t $\leq$ 3, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

$\int_0^{\sqrt{2\pi}} v(t)\,dt = -3.625 \rightarrow x\left(\sqrt{2\pi}\right) = x(0) + \int_0^{\sqrt{2\pi}} v(t)\,dt = -2.265$ Since the total distance from t = 0 to t = 3 is 4.334, the particle is still to the left of the origin at t = 3. Hence the greatest distance from the origin is 2.265. +1 $\pm$ (distance particle travels while velocity is negative), +1 answer

POW DUE 4/15/10

The use of a calculator is NOT PERMITTED on these questions. One question per side of paper.

1. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where f(3) = 25.

a. Find f " (3).

b. Write an expression for y = f(x) by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition f(3) = 25.

2. Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

a. Let y = f(x) be the particular solution to the given differential equation for 1 < x < 5 such that the line y = -2 is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum or neither at this point. Justify your answer.

b. Let y = g(x) be the particular solution to the given differential equation for –2 < x < 8, with the initial condition g(6) = -4. Find y = g(x).

3. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$

a. Find a solution y = f(x) to the differential equation satisfying $f(0) = \frac{1}{2}$

b. Find the domain and range of the function f found in part (a).