AP Calculus – AB - Problem of the Week
DUE NO LATER THAN 4/28/10 AT AP REVIEW

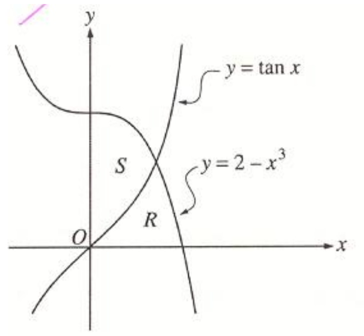
Name ______________________________
Period _____

SOLUTION TO 4/22/10 POW

One problem per side of paper.

1. ***The use of a calculator is REQUIRED on this problem.*** Let R and S be the regions in the first quadrant shown in the figure below. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

pt of intersection: $2 - x^3 = \tan x$ at $(A,B) = (0.902155, 1.265751)$



a. Find the area of R.

Area R = $\int_0^A \tan x\,dx + \int_A^{\sqrt[3]{2}} \left(2 - x^3\right)dx = 0.729$ +1 limits, +1 integrand, +1 answer

b. Find the area of S.

Area S = $\int_0^A \left(2 - x^3 - \tan x\right)dx = 1.160$ +1 limits, +1 integrand, +1 answer

c. Find the volume of the solid generated when S is revolved about the x-axis.

Volume $= \pi\int_0^A \left(\left(2 - x^3\right)^2 - \tan^2 x\right)dx = 2.652\pi$ or 8.331 +1 limits and constant, +1 integrand, +1 answer

2. ***The use of a calculator is NOT PERMITTED on this problem.*** Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} \text{ for } 0 \le x \le 3 \\ 5 - x \text{ for } 3 < x \le 5 \end{cases}$$

a. Is f continuous at x = 3? Explain why or why not.

f is continuous at x = 3 because $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 2 \to \lim_{x \to 3} f(x) = 2 = f(3)$

+1 answers yes with left hand and right hand limits, +1 explanation involving limits

b. Find the average value of f(x) on the closed interval $0 \le x \le 5$.

$\int_0^5 f(x)dx = \int_0^3 f(x)dx + \int_3^5 f(x)dx = \frac{2}{3}(x+1)^{3/2}\Big|_0^3 + \left(5x - \frac{1}{2}x^2\right)\Big|_3^5 = \frac{20}{3} \to$ Ave Value : $\frac{1}{5}\int_0^5 f(x)dx = \frac{4}{3}$

+1 $k\int_0^3 f(x)dx + k\int_3^5 f(x)dx$, +1 antiderivative of $\sqrt{x+1}$ +1 antiderivative of 5 - x, +1 evaluation and answer

c. Suppose the function g is defined by $g(x) = \begin{cases} k\sqrt{x+1} \text{ for } 0 \le x \le 3 \\ mx + 2 \text{ for } 3 < x \le 5 \end{cases}$, where k and m are constants. If g is differentiable at x = 3, what are the values of k and m?

Since g is continuous at x = 3, 2k = 3m + 2, $g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} \text{ for } 0 < x < 3 \\ m \quad \text{ for } 3 < x < 5 \end{cases}$ $\lim_{x \to 3^-} g'(x) = \frac{k}{4}$ and $\lim_{x \to 3^+} g'(x) = m$.

Since the limits exist and g is differentiable at 3, the two limits are equal.

Thus $\frac{k}{4} = m$ $\quad 8m = 3m + 2 \to m = \frac{2}{5} \to k = \frac{8}{5}$ +1 2k = 3m + 2, +1 k/4 = m, +1 values for k and m

3. ***The use of a calculator is REQUIRED on this problem.*** Let f be a function with f(1) = 4 such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$

a. Find the slope of the graph of f at the point where x = 1.

$\dfrac{dy}{dx}=\dfrac{3x^2+1}{2y}\rightarrow\dfrac{dy}{dx}\Big|_{(1,4)}=\dfrac{3+1}{2\cdot4}=\dfrac{1}{2}$ +1 answer

b. Write an equation for the line tangent to the graph of f at x = 1 and use it to approximate f(1.2).

$y-4=\frac{1}{2}(x-1)\rightarrow f(1.2)\approx4.1$ +1 equation of tangent line, +1 uses equation to approximate f(1.2)

c. Find f(x) by solving the separable differential equation $\dfrac{dy}{dx}=\dfrac{3x^2+1}{2y}$ with the initial condition f(1) = 4.

$2ydy=\left(3x^2+1\right)dx$	+1 separates variables
$y^2=x^3+x+C$	+1 antiderivative of dy term, +1 antiderivative of dx term
$4^2=1^3+1+C\rightarrow C=14$	+1 uses (1, 4) to pick one function out of family of functions
$f(x)=\sqrt{x^3+x+14}$	+1 solves for y

Note: max 0/5 if no separation of variables

Note: max 1/5 if substitutes value(s) for x,y,or $\dfrac{dy}{dx}$ before antidifferentiation

d. Use your solution from part (c) to find f(1.2).

$f(1.2)=\sqrt{1.2^3+1.2+14}\approx4.114$ +1 answer from student's solution to part c

POW DUE 4/28/10

The use of a calculator is NOT PERMITTED on these questions. One problem per side of paper.

1. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$

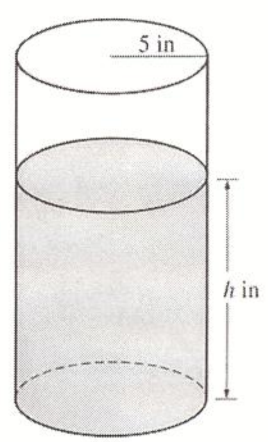
a. Find the acceleration of the particle at time t = 3.

b. Is the speed of the particle increasing at time t= 3? Given a reason for your answer.

c. Find all values of t at which the particle changes direction. Justify your answer.

d. Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

2. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure below. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$)



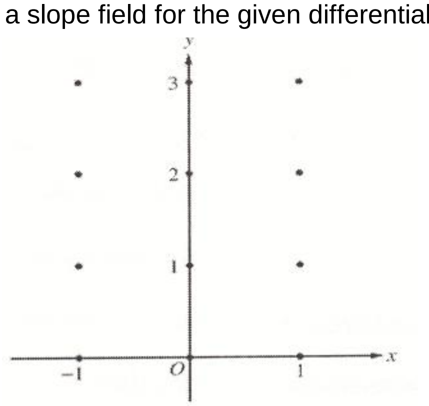
a. Show that $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$

b. Given that h = 17 at time t = 0, solve the differential equation $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$ for h as a function of t.

c. At what time t is the coffeepot empty?

3. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$

a. Recopy the axes provided and sketch a slope field for the given differential equation at the twelve points indicated.



b. While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

c. Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = 3.