AP Calculus – AB - Problem of the Week
SOLUTIONS TO 4/28/10 POW

Name ______________________________
Period _____

The use of a calculator is NOT PERMITTED on these questions. One problem per side of paper.

1. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$

a. Find the acceleration of the particle at time t = 3.

$a(t) = v'(t) = -e^{1-t} \to a(3) = -e^{-2}$ +1 v'(t), +1 a(3)

b. Is the speed of the particle increasing at time t= 3? Given a reason for your answer.

$a(3) < 0$. $v(3) = -1 + e^{-2} < 0$. Speed is increasing since v(3) < 0 and a(3) < 0. +1 answer with reason

c. Find all values of t at which the particle changes direction. Justify your answer

$v(t) = 0$ when $1 = e^{1-t} \to t = 1 \to v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$.

Therefore, the particle changes direction at t = 1.

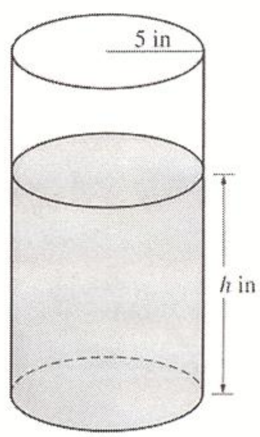
+1 solves v(t) = 0 to get t = 1, +1 justifies change in direction at t = 1

d. Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

$$\text{distance} = \int_0^3 |v(t)|\,dt = \int_0^1 \left(-1 + e^{1-t}\right)dt + \int_1^3 \left(1 - e^{1-t}\right)dt = \left(-t - e^{1-t}\right)\Big|_0^1 + \left(t + e^{1-t}\right)\Big|_1^3 = e + e^{-2} - 1$$

+1 limits, +1 integrand, +1 antidifferentiation, +1 evaluation

2. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure below. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$)



a. Show that $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$

$V = 25\pi h \rightarrow \frac{dV}{dt} = 25\pi \frac{dh}{dt} = -5\pi\sqrt{h} \rightarrow \frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = \frac{-\sqrt{h}}{5}$ +1 $\frac{dV}{dt} = -5\pi\sqrt{h}$ +1 computes $\frac{dV}{dt}$ +1 shows result

b. Given that h = 17 at time t = 0, solve the differential equation $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$ for h as a function of t.

$\frac{1}{\sqrt{h}} dh = \frac{-1}{5} dt$ +1 separates varaibles

$2\sqrt{h} = \frac{-t}{5} + C$ +1 antiderivatives

$2\sqrt{17} = 0 + C \rightarrow C = 2\sqrt{17}$ +1 constant of integration

+1 uses intial condition h = 17 when t = 0

$h = \left(\frac{-t}{10} + \sqrt{17}\right)^2$ +1 solves for h

Note: max 2/5 if no constant of integration

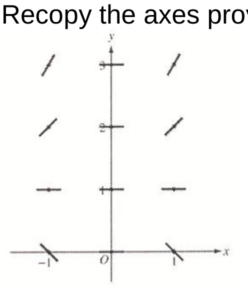
Note: 0/5 if no separation of variables

c. At what time t is the coffeepot empty?

$\left(\frac{-t}{10} + \sqrt{17}\right)^2 = 0 \rightarrow t = 10\sqrt{17}$ +1 answer

3. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$

a. Recopy the axes provided and sketch a slope field for the given differential equation at the twelve points indicated.



+1 zero slope at each point (x, y) where x = 0 or y = 1

+1 pos slope at each point (x, y) where x is not 0 and y > 1
neg slope at each point (x, y) where x is not 0 and y <1

b. While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

Slopes are positive at points (x, y) where x is not 0 and y > 1 +1 description

c. Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = 3.

$\frac{1}{y-1} dy = x^2 dx$ +1 separates variables

$\ln|y-1| = \frac{1}{3}x^3 + C$ +2 antiderivatives

$|y-1| = e^C e^{x^3/3} \rightarrow K = \pm e^C \rightarrow K = 2$ +1 constant of integration

+1 uses initial condition

$y = 1 + 2e^{x^3/3}$ +1 solves for y

Note: max 3/6 if no constant of integration

Note: 0/6 if no separation of variables