APCBC Problem of the Week Name ______________________________
DUE NO LATER THAN 12/11/09 AT 8:00 AM

SOLUTIONS FOR POW DUE 12/4/09

1. (Calculator Required) The number of gallons, P(t), of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time t = 0. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time t = 9? Why or why not?

$P'(9) = 0 - 3e^{-0.6} = -0.646 < 0$ so the amt is not increasing at this time +1 answer with reason

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

$P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0 \rightarrow t = (5\ln 3)^2 = 30.174 \rightarrow P'(t)$ is neg for $0 < t < (5\ln 3)^2$ and pos for $t > (5\ln 3)^2$.

Therefore, there is a minimum at $t = (5\ln 3)^2$. +1 sets P ' (t) = 0, +1 solves for t, +1 justification

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

$$P(30.174) = 50 + \int_0^{30.174} \left(1 - e^{-0.2\sqrt{t}}\right) dt = 35.104 < 40 \text{ so the lake is safe}$$

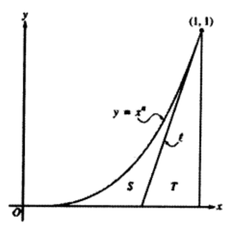
+1 integrand, +1 limits, +1 conclusion with reason based on integral of P ' (t)

(d) An investigator uses the tangent line approximate to P(t) at t = 0 as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

P ' (0) = 1 – 3 = -2. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when t = 5.

+1 slope of tangent line, +1 answer

2. (No Calculator) Let l be the line tangent to the graph of $y = x^n$ at the point (1, 1), where n > 1, as shown in the figure below.



(a) Find $\int_0^1 x^n dx$ in terms of n.

$$\int_0^1 x^n dx = \frac{x^{n+1}}{n+1}\Big|_0^1 = \frac{1}{n+1}$$ +1 antiderivative of x^n, +1 answer

(b) Let T be the triangular region bounded by l, the x-axis, and the line x = 1. Show that the area of T is $\frac{1}{2n}$.

Let b be the length of the base of triangle T. 1/b is the slope of the line l, which is n… $\text{Area}(T) = \frac{1}{2}b(1) = \frac{1}{2n}$

+1 slope of line l is n, +1 base of T is 1/n, +1 shows area is $\frac{1}{2n}$

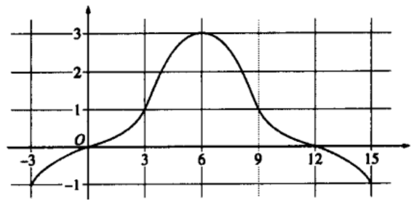
(c) Let S be the region bounded by the graph of $y = x^n$, the line l, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the error of S.

$$\text{Area}(S) = \int_0^1 x^n dx - \text{Area}(T) = \frac{1}{n+1} - \frac{1}{2n}$$

$$\frac{d\text{Area}(S)}{dn} = \frac{-1}{(n+1)^2} + \frac{1}{2n^2} = 0 \rightarrow 2n^2 = (n+1)^2 \rightarrow n = \frac{1}{\sqrt{2}-1} = 1 + \sqrt{2}$$

+1 area of S in terms of n, +1 derivative, +1 sets derivative equal to 0, +1 solves for n

3. (No Calculator) The graph of a differentiable function f on the closed interval [-3, 15] is shown in the figure below. The graph of f has a horizontal tangent line at x = 6. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of *f*

(a) Find g(6), g ' (6), and g " (6).

$g(6) = 5 + \int_6^6 f(t)\,dt = 5 \rightarrow g'(6) = f(6) = 3 \rightarrow g''(6) = f'(6) = 0$ +1 g(6), +1 g ' (6), +1 g " (6)

(b) On what intervals is g decreasing? Justify your answer.

g is decreasing on [-3, 0] and [12, 15] since $g'(x) = f(x)$ for $x < 0$ and $x > 12$ +1 [-3, 0], +1 [12, 15], +1 justification

(c) On what intervals is the graph of g concave down? Justify your answer.

The graph of g is concave down on (6, 15) since g ' = f is decreasing on this interval +1 interval, +1 justification

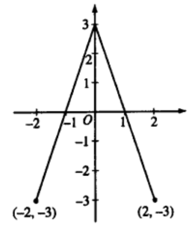
(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

$\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1) = 12$ +1 trapezoidal method

PROBLEM OF THE WEEK DUE 12/11/09

1. (**no calculator**) The graph of the function f shown below consists of two line segments. Let g be the function given by

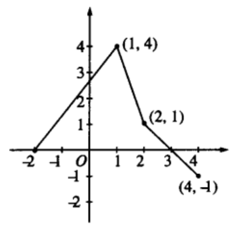
$$g(x)=\int_0^x f(t)\,dt.$$



Graph of f

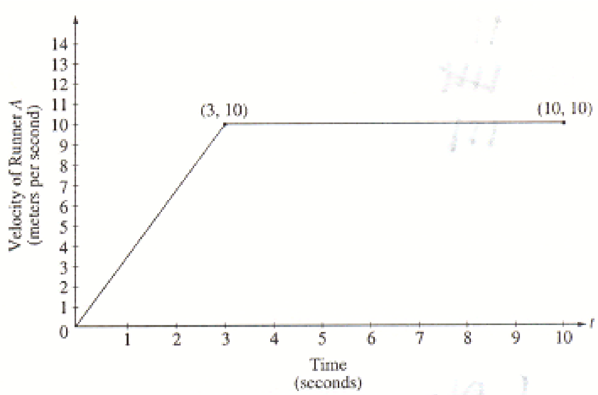
(a) Find g(-1), g ' (-1), and g " (-1).
(b) For what values of x in the open interval (-2, 2) is g increasing? Explain your reasoning.
(c) For what values of x in the open interval (-2, 2) is the graph of g concave down? Explain your reasoning.
(d) Sketch the graph of g on the closed interval [-2, 2].

2. (**no calculator**) The graph of the function f, consisting of three line segments, is given below. Let $g(x)=\int_1^x f(t)\,dt$.



(a) Computer g(4) and g(-2).
(b) Find the instantaneous rate of change of g, with respect to x, at x = 1.
(c) Find the absolute minimum value of g on the closed interval [-2, 4]. Justify your answer.
(d) The second derivative of g is not defined at x = 1 and x = 2. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

3. (**calculator required**) Two runners, A and B, run on a straight racetrack for $0 < t \le 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of runner B is given by the function v defined by $v(t)=\dfrac{24t}{2t+3}$



(a) Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure
(b) Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.
(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.