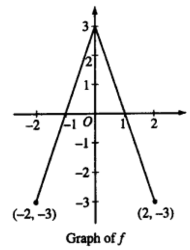
APCBC Problem of the Week Name ______________________________

SOLUTIONS TO POW 12/11/09

1. (**no calculator**) The graph of the function f shown below consists of two line segments. Let g be the function given by

$g(x)=\int_0^x f(t)\,dt$.



(a) Find g(-1), g ‘ (-1), and g “ (-1).

$g(-1)=\int_0^{-1} f(t)\,dt=\frac{-3}{2},\ g'(1)=f(-1)=0,\ g''(1)=f'(-1)=3,\ +1\,g(-1), +1\ g'(1),\ +1\,g''(1)$

(b) For what values of x in the open interval (-2, 2) is g increasing? Explain your reasoning.

g is increasing on $-1<x<1$ because $g'(x)=f(x)>0$ on this interval +1 interval, +1 reason

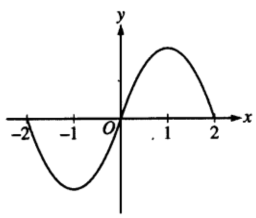
(c) For what values of x in the open interval (-2, 2) is the graph of g concave down? Explain your reasoning.

The graph of g is concave down on 0 < x < 2 because $g''(x)=f(x)>0$ on this interval

OR +1 interval, +1 reason

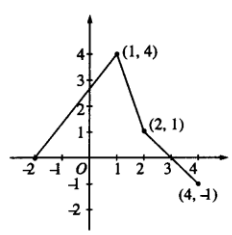
because $g'(x)=f(x)$ is decreasing on this interval

(d) Sketch the graph of g on the closed interval [-2, 2].



+1 G(-2) = G(0) = G(2) = 0, +1 appropriate inc/dec and concavity <-1> vertical asymptote

2. (**no calculator**) The graph of the function f, consisting of three line segments, is given below. Let $g(x)=\int_1^x f(t)\,dt$.



(a) Computer g(4) and g(-2).

$g(4)=\int_1^4 f(t)\,dt=\frac{3}{2}+1+\frac{1}{2}-\frac{1}{2}=\frac{5}{2},\ g(-2)=\int_1^{-2} f(t)\,dt=\frac{-1}{2}(12)=-6\ +1\,g(4),\ +1\,g(-2)$

(b) Find the instantaneous rate of change of g, with respect to x, at x = 1.

$g'(1)=f(1)=4$ +1 answer

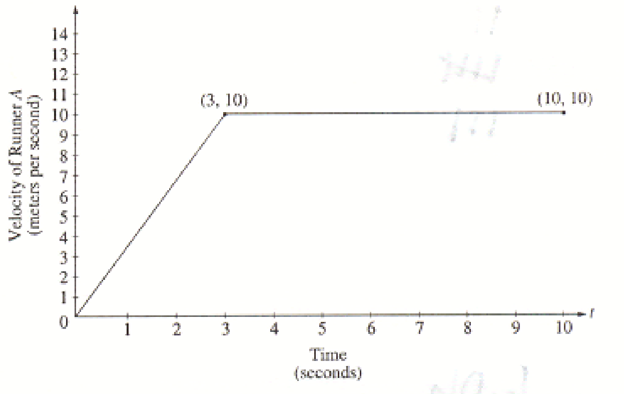
(c) Find the absolute minimum value of g on the closed interval [-2, 4]. Justify your answer.

g is increasing on [-2, 3] and decreasing on [3, 4]. Therefore, g has absolute minimum at an endpoint of [-2, 4]. Since g(-2) = -6 and g(4) = ½, the absolute minimum value is –6. +1 interior analysis, +1 endpoint analysis, +1 answer

(d) The second derivative of g is not defined at x = 1 and x = 2. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

One; x = 1, On (-2, 1) g " (x) = f ' (x) > 0. On (1, 2), g " (x) = f ' (x) < 0. On (2, 4), g " (x) = f ' (x) < 0. Therefore (1, g(1)) is a point of inflection and (2, g(2)) is not. +1 choice of x = 1 only, +1 shows (1, g(1)) is a pt of inflection, +1 shows (2, g(2)) is not a point of inflection.

3. (**calculator required**) Two runners, A and B, run on a straight racetrack for $0 < t \leq 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of runner B is given by the function v defined by $v(t) = \frac{24t}{2t+3}$



(a) Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure

Runner A: velocity = $\frac{10}{3} \cdot 2 = \frac{20}{3} = 6.667\ m/s$ Runner B : $v(2) = \frac{48}{7} = 6.857\ m/s$

+1 velocity of Runner A, +1 velocity of Runner B

(b) Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.

Runner A: acceleration -= $\frac{10}{3} = 3.333\ m/s^2$ Runner B : $a(2) = v'(2) = \frac{72}{(2t+3)^2} |_{t=2} = \frac{72}{49} = 1.469\ m/s^2$

+1 acceleration for Runner A, +1 acceleration for Runner B

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

Runner A: distance = $\frac{1}{2}(3)(10) + 7(10) = 85\ m$ Runner B distance = $\int_0^{10} \frac{24t}{2t+3} dt = 83.336\ m$

+2 distance for Runner A (+1 method, +1 answer), +2 distance for Runner B (+1 integral, +1 answer)
+1 units in parts a, b, and c