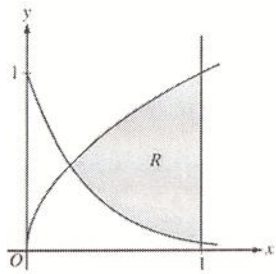
APCBC Problem of the Week — Name ________________________________

DUE NO LATER THAN 1/22/10 AT 8:00 AM

SOLUTIONS TO 1/15/10 POW

The use of a calculator is REQUIRED on these questions. One problem per side of paper.

1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line x = 1, as shown in the figure below.



Pt of Int $\rightarrow$ $e^{-3x} = \sqrt{x}$ at (T, S) = (0.238734, 0.488604) +1 correct limits in integral is a, b, or c

a. Find the area of R.

$\text{Area} = \int_T^1 \left(\sqrt{x} - e^{-3x}\right)dx = 0.442$ +1 integrand, +1 answer

b. Find the volume of the solid generated when R is revolved about the horizontal line y = 1.

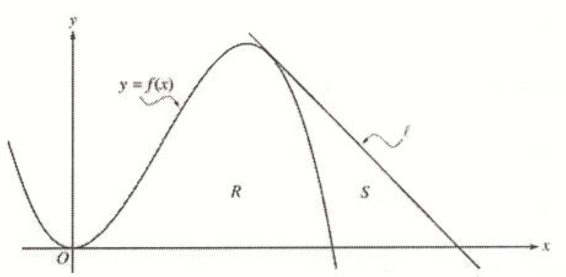
$\text{Volume} = \pi\int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right)dx = 0.453\pi$ or 1.423 +2 integrand, +1 answer

c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

$\text{Length} = \sqrt{x} - e^{-3x} \rightarrow \text{Height} = 5\left(\sqrt{x} - e^{-3x}\right) \rightarrow \text{Volume} = \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

+2 integrand, +1 answer

2. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line y = 18 – 3x, where ℓ is tangent to he graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis as shown below.



a. Show that ℓ is tangent to the graph of y = f(x) at the point x = 3.

$f'(x) = 8x - 3x^2, f(3) = -3, f(3) = 9 \rightarrow y - 9 = -3(x - 3)$ which is the equation of ℓ.

+1 f'(3) and f(3), +1 eq of tan line

b. Find the area of S.

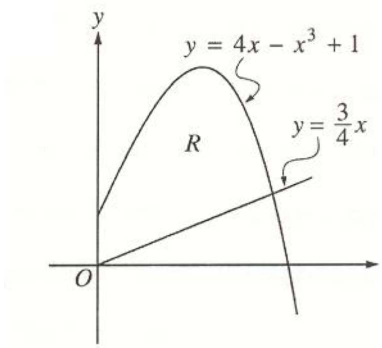
$f(x) = 0$ at x = 4; The line interesects the x-axis at x = 6. Area = $\frac{1}{2}(3)(9) - \int_3^4 \left(4x^2 - x^3\right)dx = 7.916$

+1 limits, +1 integrand, +1 area of triangular region, +1 answer

c. Find the volume of the solid generated when R is revolved about the x-axis.

$\text{Volume} = \pi\int_0^4 \left(4x^2 - x^2\right)^2 dx = 156.038\pi$ or 490.208 +1 limits and constant, +1 integrand, +1 answer

3. Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$



$4x - x^3 + 1 = \frac{3}{4}x \rightarrow x = 1.94045 = A$

a. Find the area of R

$\text{Area} = \int_0^A \left(4x - x^3 + 1 - \frac{3}{4}x\right)dx = 4.514$ +1 limits, +1 integrand, +1 answer

b. Find the volume of the solid generated when R is revolved about the x-axis.

$\text{Volume} = \pi\int_0^A \left(\left(4x - x^3 + 1\right)^2 - \left(\frac{3}{4}x\right)^2\right)dx = 18.291\pi$ or 57.463 +1 limits and constant, +1 integrand, +1 answer

c. Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

$\text{Perimeter} = 1 + \sqrt{(1.940)^2 + (1.455)^2} + \int_0^A \sqrt{1 + \left(4 - 3x^2\right)^2}\,dx$

+1 uses $y' = 4 - 3x^2$ in integrand, +1 arc length integral, +1 answer

POW DUE 1/22/10

One problem per side of paper.

1. ***The use of a calculator is REQUIRED for this problem.*** A blood vessel is 360 millimeters long with circular cross sections of varying diameter. The table below gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and B(x) is a twice differentiable function that represents the diameter at that point.

Distance x(mm)	0	60	120	180	240	300	360
Diameter B(x) (mm)	24	30	28	30	26	24	26

a. Write an integral expression in terms of B(x) that represents the average radius, in mm, of the blood vessel between x = 0 and x = 360.

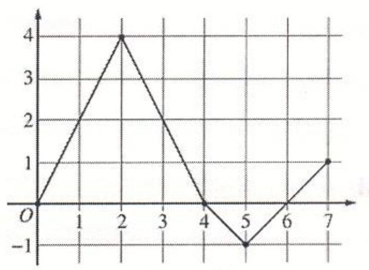
b. Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

c. Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

d. Explain why there must be at least one value x, for 0 < x < 360, such that B " (x) = 0.

2. ***The use of a graphing calculator is NOT PERMITTED on this problem.*** Let f be a function defined on the closed interval [0, 7]. The graph of f, consisting of four line segments, is shown below. Let g be the function given by

$g(x)=\int_{2}^{x} f(t)dt$.



Graph of *f*

(a) Find g(3), g ' (3), and g " (3).
(b) Find the average rate of change of g on the interval 0 ≤ x ≤ 3.
(c) For how many values c, where 0 < c < 3, is g ' (c) equal to the average rate found in part (b)? Explain your reasoning.
(d) Find the x-coordinate of each point of inflection of the graph of g on the interval 0 < x < 7. Justify your answer.

3. ***The use of a calculator is REQUIRED for this problem.*** A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) for 0 ≤ t ≤ 40are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

a. Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_{0}^{40} v(t)dt$.

Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_{0}^{40} v(t)dt$ in terms of the plane's flight.

b. Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer.

c. The function f, defined by $f(t)=6+\cos\frac{t}{10}+3\sin\frac{7t}{40}$, is used to model the velocity of the plane, in miles per minute, for 0 ≤ t ≤ 40. According to this model, what is the acceleration of the plane at t = 23? Indicate units of measure.

d. According to the model f, given in part (c) , what is the average velocity of the plane, in miles per minute, over the time interval 0 ≤ t ≤ 40?