APCBC Problem of the Week   Name ______________________________

DUE NO LATER THAN 1/29/10 AT 8:00 AM

SOLUTIONS TO 1/22/09 POW

One problem per side of paper.

1. ***The use of a calculator is REQUIRED for this problem.*** A blood vessel is 360 millimeters long with circular cross sections of varying diameter. The table below gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and B(x) is a twice differentiable function that represents the diameter at that point.

Distance x(mm)	0	60	120	180	240	300	360
Diameter B(x) (mm)	24	30	28	30	26	24	26

a. Write an integral expression in terms of B(x) that represents the average radius, in mm, of the blood vessel between x = 0 and x = 360.

$\frac{1}{360}\int_0^{360}\frac{B(x)}{2}dx$ +1 limits and constant, +1 integrand

b. Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

$\frac{1}{360}\left[120\left(\frac{B(60)}{2}+\frac{B(180)}{2}+\frac{B(300)}{2}\right)\right]=14$ +1 B(60) + B(180) + B(300), +1 answer

c. Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

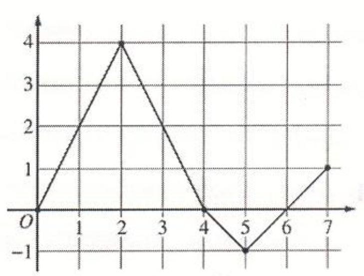
$\frac{B(x)}{2}$ is the radius, so $\pi\left(\frac{B(x)}{2}\right)^2$ is the area of the cross section at x. The expression is the volume in mm^3 of the blood vessel between 125 and 275 from the end of the vessel

+1 volume in cubic mm, +1 between x = 125 and x = 275

d. Explain why there must be at least one value x, for 0 < x < 360, such that B " (x) = 0.

By the MVT, $B'(c_1)=0$ for some c_1 in (60, 180) and $B'(c_2)=0$ for some c_2 in (240, 360). The MVT applied to $B'(x)$ shows that $B''(x)=0$ for some x in the interval (c_1, c_2). +2 explains why there are two values of x where $B'(x)$ has the same value, +1 explains why that means $B''(x)=0$ for some $0<x<360$

2. ***The use of a graphing calculator is NOT PERMITTED on this problem.*** Let f be a function defined on the closed interval [0, 7]. The graph of f, consisting of four line segments, is shown below. Let g be the function given by $g(x)=\int_2^x f(t)dt$.



Graph of *f*

(a) Find g(3), g ' (3), and g " (3).

$g(3)=\int_2^3 f(t)dt=\frac{1}{2}(4+2)=3, g'(3)=f(3)=2, g''(3)=f'(3)=\frac{0-4}{4-2}=-2$ +1 g(3), +1 g'(3), +1 g"(3)

(b) Find the average rate of change of g on the interval 0 ≤ x ≤ 3.

$\frac{g(3)-g(0)}{3-0}=\frac{1}{3}\int_0^3 f(t)dt=\frac{1}{3}\left(\frac{1}{2}(2)(4)+\frac{1}{4}(4+2)\right)=\frac{7}{3}$ +1 $g(3)-g(0)=\int_0^3 f(t)dt$, +1 answer

(c) For how many values c, where 0 < c < 3, is g ‘ (c) equal to the average rate found in part (b)? Explain your reasoning.

There are two values of c. We need $\frac{7}{3} = g'(c) = f(c)$. The graph of f intersects the line y = $\frac{7}{3}$ at two places between 0 and 3. +1 answer of 2, +1 reason, 1/2 if answer is 1 by MVT

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval 0 < x < 7. Justify your answer.

x = 2 and x = 5 because g ' = f changes from inc to dec at x = 2, and from dec to inc at x = 5.

+1 x = 2 and x = 5 only, +1 justification (ignore discussion at x = 4)

3. ***The use of a calculator is REQUIRED for this problem.*** A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) for $0 \le t \le 40$are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

a. Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$.

Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane’s flight.

Midpt Sum = $10[9.2 + 7.0 + 2.4 + 4.3] = 229$. The integral gives the total distance in miles that the plane flies during the 40 minutes. +1 $9.2 + 7.0 + 2.4 + 4.3$, +1 answer, +1 meaning with units

b. Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer.

By the Mean Value Theorem, v ‘ (t) = 0 somewhere in the interval (0, 15) and somewhere in the interval (25, 30). Therefore the acceleration will equal 0 for at least two values of t +1 two instances, +1 justification

c. The function f, defined by $f(t) = 6 + \cos\frac{t}{10} + 3\sin\frac{7t}{40}$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at t = 23? Indicate units of measure.

$f'(23) = -0.407$ miles per minute^2 +1 answer with units

d. According to the model f, given in part (c) , what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

$ave_{velocity} = \frac{1}{40}\int_0^{40} f(t)\,dt = 5.916$ +1 limits, +1 integrand, +1 answer

POW DUE 1/29/10

1. ***The use of a calculator is REQUIRED for this problem.*** Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$

a. Find the area of the region enclosed by the graphs of f and g between x = ½ and x = 1.

b. Find the volume of the solid generated when the region enclosed by the graphs of f and g between x = ½ and x = 1 is revolved about the line y = 4.

c. Let h be the function given by h(x) = f(x) – g(x). Find the absolute minimum value of h(x) on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of h(x) on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answer.

2. ***The use of a calculator is REQUIRED for this problem.*** Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4\sin\frac{t}{2}$ for $0 \le t \le 30$, where F(t) is measured in cars per minute and t is measured in minutes.

a. To the nearest whole number, how many cars pass through the intersection over the 30-minutes period?

b. Is the traffic flow increasing or decreasing at t = 7? Give a reason for your answer.

c. What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

d. What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

3. ***The use of a calculator is NOT PERMITTED for this problem.*** Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

a. Find the average value of g on the closed interval [1, 4].

b. Let S be the solid generated when the region bounded by the graph of y = g(x), the vertical lines x = 1 and x = 4, and the x-axis is revolved about the x-axis. Find the volume of S.

c. For the solid S, given in part (b), find the average value of the areas of the cross sections perpendicular to the x-axis.

d. The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b\to\infty} \frac{\int_a^b f(x)\,dx}{b-a}$. Show that the improper integral $\int_4^\infty g(x)\,dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.