APCBC Problem of the Week Name ________________________________

DUE NO LATER THAN 2/5/10 AT 8:00 AM

SOLUTIONS TO 1/29/10 POW

1. ***The use of a calculator is REQUIRED for this problem.*** Let f and g be the functions given by

$f(x) = e^x$ and $g(x) = \ln x$

a. Find the area of the region enclosed by the graphs of f and g between x = ½ and x = 1.

Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ +1 integral, +1 answer

b. Find the volume of the solid generated when the region enclosed by the graphs of f and g between x = ½ and x = 1 is revolved about the line y = 4.

Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2 \right) dx = 7.515\pi$ or 23.609

+1 limits and constant, +2 integrand <-1> each error, +1 answer

c. Let h be the function given by h(x) = f(x) – g(x). Find the absolute minimum value of h(x) on the closed interval ½ $\le$ x $\le$ 1, and find the absolute maximum value of h(x) on the closed interval ½ $\le$ x $\le$ 1. Show the analysis that leads to your answer.

$h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0 \rightarrow x = 0.567143$ +1 considers h ' (x) = 0

Abs min and abs max occur at critical point or at endpoints +1 identifies critical pt and endpts as candidates

$h(0.567143) = 2.330,\ h(0.5) = 2.3418,\ h(1) = 2.718$ +1 answers

Absolute min is 2.330, absolute max is 2.718 Errors in computation come off third point

2. ***The use of a calculator is REQUIRED for this problem.*** Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4\sin\frac{t}{2}$ for $0 \le t \le 30$, where F(t) is measured in cars per minute and t is measured in minutes.

a. To the nearest whole number, how many cars pass through the intersection over the 30-minutes period?

$\int_{0}^{30} F(t)\,dt = 2474$ cars +1 limits, +1 integrand, +1 answer

b. Is the traffic flow increasing or decreasing at t = 7? Give a reason for your answer.

$F'(7) = -1.872$ Since F ' (7) < 0, the traffic flow is decreasing at t = 7. +1 limits, +1 integrand, +1 answer

c. What is the average value of the traffic flow over the time interval 10 $\le$ t $\le$ 15? Indicate units of measure.

$\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min +1 limits, +1 integrand, +1 answer

d. What is the average rate of change of the traffic flow over the time interval 10 $\le$ t $\le$ 15? Indicate units of measure.

$\frac{F(15) - F(10)}{15 - 10} = 1.517\ \text{cars/min}^2$ +1 answer

+1 units in parts b and c

3. ***The use of a calculator is NOT PERMITTED for this problem.*** Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

a. Find the average value of g on the closed interval [1, 4].

$\frac{1}{3}\int_{1}^{4} \frac{1}{\sqrt{x}}\,dx = \frac{1}{3}\cdot 2\sqrt{x}\,\Big|_1^4 = \frac{2}{3}$ +1 integral, +1 antidifferentiaion and evaluation

b. Let S be the solid generated when the region bounded by the graph of y = g(x), the vertical lines x = 1 and x = 4, and the x-axis is revolved about the x-axis. Find the volume of S.

Volume $= \pi\int_{1}^{4} \frac{1}{x}\,dx = \pi \ln x\,\Big|_1^4 = \pi \ln 4$ +1 integral, +1 antidifferentiation and evaluation

c. For the solid S, given in part (b), find the average value of the areas of the cross sections perpendicular to the x-axis.

The cross section at x has area $\pi\left(\frac{1}{\sqrt{x}}\right)^2 = \frac{\pi}{x}$. $\text{Value}_{ave} = \frac{1}{3}\int_{1}^{4} \frac{\pi}{x}\,dx = \frac{1}{3}\pi \ln 4$ +1 answer

d. The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b\to\infty} \frac{\int_a^b f(x)\,dx}{b-a}$. Show that the improper integral $\int_4^{\infty} g(x)\,dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.

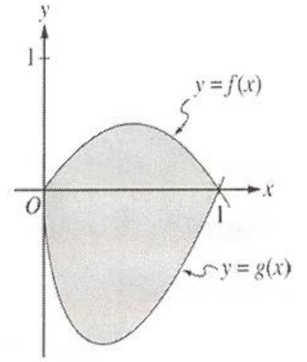
$\int_4^{\infty} g(x)\,dx = \lim_{b\to\infty} \int_4^{\infty} \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\left(2\sqrt{b}-4\right) = \infty$. The limit is not finite, so divergent. $\frac{\int_4^b g(x)\,dx}{b-4} = \frac{1}{b-4}\int_4^b \frac{1}{\sqrt{x}}\,dx = \frac{2\sqrt{b}-4}{b-4}$

$\lim_{b\to\infty} \frac{2\sqrt{b}-4}{b-4} = 0$ +1 $\int_4^b g(x)\,dx = 2\sqrt{b}-4$, +1 indicates integral diverges, +1 $\frac{1}{b-4}\int_4^b g(x)\,dx = \frac{2\sqrt{b}-4}{b-4}$, +1 finite limit as $b \to \infty$

POW DUE 2/5/10

The use of a calculator is REQUIRED on these questions.

1. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure below.



a. Find the area of the shaded region enclosed by the graphs of f and g.
b. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line y = 2.
c. Let h be the function given by h(x) = kx(1 – x) for $0 \le x \le 1$. For each k > 0, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

2. The temperature, in degrees Celsius, of the water in a pond is a differentiable function W of time t. The table below shows the water temperature as recorded every three days over a fifteen day period.

t (days)	0	3	6	9	12	15
W(t) (Celsius)	20	31	28	24	22	21

a. Use data from the table to find an approximation for W ‘ (12). Show the computations that lead to your answer. Indicate units of measure.
b. Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
c. A student proposes the function P, given by $P(t) = 20 + 10te^{-t/3}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and P(t) is measured in degrees Celsius. Find P ‘ (12). Using appropriate units, explain the meaning of your answer in terms of water temperature.
d. Use the function P defined in part (c) to find the average value, in degrees Celsius, of P(t) over the time interval $0 \le t \le 15$ days.

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured every three hours for a 24-hour period.

t	0	3	6	9	12	15	18	21	24
R(t)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

a. Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.
b. Is there some time t, 0 < t < 24, such that R ‘ (t) = 0? Justify your answer.
c. The rate of water flow R(t) can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use Q(t) to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.