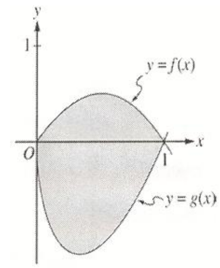
APCBC Problem of the Week Name ________________________________

DUE NO LATER THAN 2/12/10 AT 8:00 AM

SOLUTIONS TO 2/5/10 POW

1. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure below.



a. Find the area of the shaded region enclosed by the graphs of f and g.

$$\text{Area} = \int_0^1 (f(x) - g(x))\,dx = \int_0^1 \left(2x(1-x) - 3(x-1)\sqrt{x}\right)dx = 1.133$$ +1 integral, +1 answer

b. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line y = 2.

$$\text{Volume} = \pi\int_0^1 \left((2-g(x))^2 - (2-f(x))^2\right)dx = 16.179$$ +1 limits and constant, +2 integrand <-1> each error, +1 answer

c. Let h be the function given by h(x) = kx(1 – x) for $0 \le x \le 1$. For each k > 0, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

$$\text{Volume} = \int_0^1 (h(x) - g(x))^2\,dx = 15$$ +2 integrand, +1 answer

2. The temperature, in degrees Celsius, of the water in a pond is a differentiable function W of time t. The table below shows the water temperature as recorded every three days over a fifteen day period.

t (days)	0	3	6	9	12	15
W(t) (Celsius)	20	31	28	24	22	21

a. Use data from the table to find an approximation for W ' (12). Show the computations that lead to your answer. Indicate units of measure.

Difference quotient, e.g. $W'(12) \approx \dfrac{W(15) - W(9)}{15-9} = \dfrac{-1}{2}\,°C/\text{day}$

b. Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

$\dfrac{3}{2}(20 + 2(31 + 28 + 24 + 22) + 21) = 376.5$ $\text{Temp}_{ave} \approx \dfrac{1}{15}(376.5) = 25.1°C$ +1 trapezoidal method, +1 answer

c. A student proposes the function P, given by $P(t) = 20 + 10te^{-t/3}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and P(t) is measured in degrees Celsius. Find P ' (12). Using appropriate units, explain the meaning of your answer in terms of water temperature.

$P'(12) = 10e^{-t/3} - \dfrac{10}{3}te^{-t/3}\Big|_{t=12} = -30e^{-4} = -0.549°C/\text{day}$. This means that the temperature is decreasing at the rate of $-0.549°C/\text{day}$ when t = 12 days. +1 P ' (12), +1 interpretation

d. Use the function P defined in part (c) to find the average value, in degrees Celsius, of P(t) over the time interval $0 \le t \le 15$ days.

$$\frac{1}{15}\int_0^{15}\left(20 + 10te^{-t/3}\right)dt = 25.757°C$$ +1 integrand, +1 limits and average value constant, +1 answer

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured every three hours for a 24-hour period.

t	0	3	6	9	12	15	18	21	24
R(t)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

a. Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

$\int_0^{24} R(t)\,dt \approx 6[10.4+11.2+11.3+10.2] = 258.6$ gallons. This is an approximation to the total flow in galons of water from the pipe in the 24 hour period. +1 [10.4+11.2+11.3+10.2], +1 answer, +1 explanation

b. Is there some time t, 0 < t < 24, such that R ‘ (t) = 0? Justify your answer.

Yes, Since R(0) = R(24) = 9.6, the Mean Value Theorem guarantees that there is a t, 0 < t < 24, such that R ‘ (t) = 0.
+1, answer, +1 MVT or equivalent

c. The rate of water flow R(t) can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use Q(t) to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

Ave value of Q(t) = $\frac{1}{24}\int_0^{24} \frac{1}{79}\left(768 + 23t - t^2\right)dt = 10.785$ ga/hr

+1 limits and ave value constant, +1 Q(t) as integrand,

+1 answer

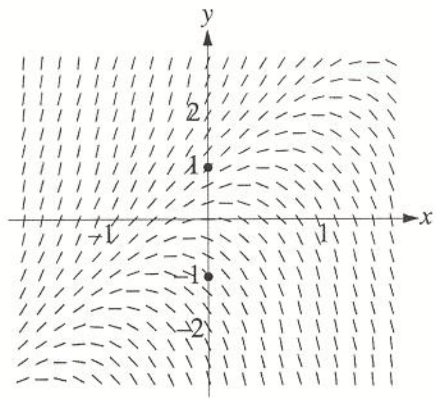
+1 units in parts a and c

POW DUE 2/12/10

1. ***The use of a calculator is REQUIRED for this problem.*** Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x) = \frac{1+e^x}{x^2}$

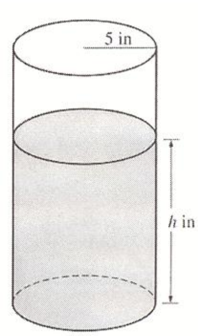
a. Write an equation of the line tangent to the graph of f at x = 3 and use it to approximate f(3.1).
b. Use Euler's method, starting at x = 3, with a step size of 0.05, to approximate f(3.1). Use f " to explain why this approximation is less than f(3.1).
c. Use $\int_3^{3.1} f'(x)\,dx$ to evaluate f(3.1)

2. ***The use of a calculator is NOT PERMITTED for this problem.*** Consider the differential equation $\frac{dy}{dx} = 2y - 4x$



a. The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point (0, 1) and sketch the solution curve that passes through the point (0, -1).
b. Let f be the function that satisfies the given differential equation with the initial condition f(0) = 1. Use Euler's method, starting at x = 0 with a step size of 0.1, to approximate f(0.2). Show the work that leads to your answer.
c. Find the value of b for which y = 2x + b is a solution to the given differential equation. Justify your answer.
d. Let g be the function that satisfies the given differential equation with the initial condition g(0) = 0. Does the graph of g have a local extremum at the point (0, 0)? If so, is the point a local maximum or a local minimum? Justify your answer.

3. ***The use of a calculator is NOT PERMITTED for this problem.*** A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure below. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$)



a. Show that $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$

b. Given that h = 17 at time t = 0, solve the differential equation $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$ for h as a function of t.

c. At what time t is the coffeepot empty?