APCBC Problem of the Week Name ________________________________
DUE NO LATER THAN 3/19/10 AT 8:00 AM

SOLUTIONS TO 3/12/10 POW

1. ***The use of a calculator is REQUIRED on this problem.*** Consider the following table of values for the differentiable function f

x	1.0	1.2	1.4	1.6	1.8
f(x)	5.0	3.5	2.6	2.0	1.5

a. Estimate f ' (1.4). Show the work that leads to your answer.

Slope of any secant line about x = 1.4, i.e. $\frac{2.0-3.5}{1.6-1.2} = -3.75$ +1 slope of secant line , +1 answer

b. Give an equation for the tangent line to the graph of f at x = 1.4.

$f(x) - 2.6 = -3.75(x-1.4)$ +1 uses slope from (a), +1 equation

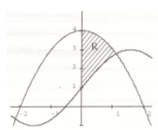
c. What is the sign of f " (1.4)? Explain your answer.

Since the slope of the curve is decreasing at a decreasing rate, the curve is concave up, or f " (1.4) > 0
+1 answer, +1 justification

d. Using the data in the table, find a midpoint approximation with 2 equal subdivisions for $\int_{1.0}^{1.8} f(x)dx$

$(0.4)(3.5+2.0) = 2.2$ +1 width 0.4, +1 heights 3.5 and 2.0, +1 answer

2. ***The use of a calculator is REQUIRED on this problem.*** Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 4 - x^2$ and $y = 1 + 2\sin x$ as shown in the figure below.



a. Find the area of R.

Point of Intersection: (1.102, 2.785) +1 point of intersection

$\int_0^A \left[\left(4-x^2\right)-\left(1+2\sin x\right)\right]dx = 1.764$ +1 limits and integrand, +1 answer

b. Find the volume of the solid generated when R is revolved about the x-axis.

$\pi\int_0^A \left[\left(4-x^2\right)^2 - \left(1+2\sin x\right)^2\right]dx = 30.460$ +1 limits and constant, +1 $\pi R^2 - \pi r^2$, +1 answer

c. Find the volume of the solid whose base is R and whose cross sections perpendicular to the x-axis are squares.

$\int_0^A \left[\left(4-x^2\right)-\left(1+2\sin x\right)\right]^2 dx = 3.671$ +1 limits, +1 integrand, +1 answer

3. ***The use of a calculator is NOT PERMITTED on this problem***. A particle moves along the x-axis in such a way that its acceleration at time t for t ≥ 0 is given by $a(t) = 6(2t+1)^{-3/2}$. When t = 4, the position of the particle is 9 and its velocity is 1.

a. Write an equation for the velocity, v(t), of the particle for all t ≥ 0.

$\int a(t)dt = v(t) = \frac{-6}{\sqrt{2t+1}} + C \rightarrow 1 = \frac{-6}{\sqrt{2(4)+1}} + C \rightarrow C = 3 \rightarrow v(t) = \frac{-6}{\sqrt{2t+1}} + 3$

+1 integrates a(t), +1 solves for C, +1 answer

b. Find the values of t for which the particle is at rest.

$\frac{-6}{\sqrt{2t+1}} + 3 = 0 \rightarrow t = \frac{3}{2}$ +1 sets v(t) = 0, +1 answer

c. Write an equation for the position, s(t), of the particle for all t ≥ 0.

$\int v(t)dt = \int\left(\frac{-6}{\sqrt{2t+1}} + 3\right)dt = -6\sqrt{2t+1} + 3t + C \rightarrow 9 = -6\sqrt{2(4)+1} + 3(4) + C \rightarrow C = 15 \rightarrow s(t) = -6\sqrt{2t+1} + 3t + 15$

+1 integrates v(t), +1 solves for C, +1 answer

d. Find the total distance traveled by the particle from t = 0 to t = 4.

From part c, $\left(-6\sqrt{2t+1} + 3t\right)\Big|_0^4 = -6 - (-6) = -12$ or 12 +1 answer

POW DUE 3/19/10

The use of a calculator is NOT PERMITTED on these problems.

1. Let f be a differentiable function such that f “ is continuous and f and f ‘ have the values given the table below

x	0	1	2	3	4	5
f(x)	1	17	3	8	9	11
f ‘ (x)	25	21	19	15	13	-2

Use the information in the table to

a. Approximate f “ (x) at x = 2.

b. Evaluate $\int_0^2 xf'\left(x^2\right)dx$

c. Evaluate $\int_1^3 xf''\left(x\right)dx$

2. Let f be the function defined by $f\left(x\right)=xe^{-kx}$, where k is a positive constant.

a. Find, in terms of k, the x-coordinate of each critical point of f.

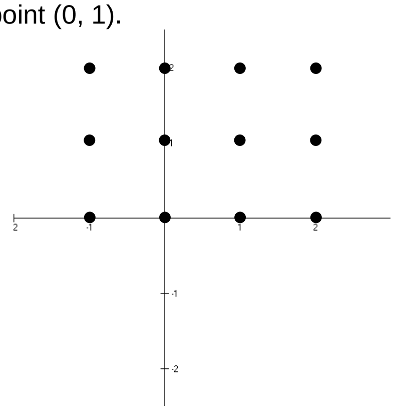
b. For each critical number x, determine whether f(x) is a relative maximum, relative minimum, or neither. Justify your answer.

c. On what interval(s) is the graph of f concave up?

d. Write an equation of the horizontal asymptote for the graph of f.

3. Consider the differential equation $\frac{dy}{dx}=2x-y$

a. On the axes below, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point (0, 1).



b. The solution curve that passes through the point (0, 1) has a local minimum at x = $\ln\frac{3}{2}$. What is the y-coordinate of this local minimum?

c. Let y = f(x) be the particular solution to the given differential equation with the initial condition f(0) = 1. Use Euler’s method, starting at x = 0 with two steps of equal size, to approximate f(-0.4). Show the work that leads to your answer.

d. Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part c is less than or greater than f(-0.4). Explain your reasoning.