APCBC Problem of the Week Name _____

DUE NO LATER THAN 4/23/10 AT 8:00 AM

SOLUTIONS TO 4/16/10 POW

The use of a calculator is REQUIRED on these problems.

1. An object moving along a curve in the xy-plane has position (x(t), y(t)) at time $t \geq 0$ with $\frac{dx}{dt} = 12t - 3t^2$ and $\frac{dy}{dt} = \ln\left(1+(t-4)^4\right)$. At time t = 0, the object is at position (-13, 5). At time t = 2, the object is at point P with x-coordinate 3.

   a. Find the acceleration vector at time t = 2 and the speed at time t = 2.

   $x''(2) = 0, y''(2) = \frac{-32}{17} \approx -1.882; a(2) = \langle 0, -1.882\rangle$; Speed $= \sqrt{12^2 + (\ln 17)^2} = 12.329$ or 12.330

   +1 acceleration vector, +1 speed

   b. Find the y-coordinate of P.

   $y(t) = y(0) + \int_0^t \ln\left(1+(u-4)^4\right)du \rightarrow y(2) = 5 + \int_0^2 \ln\left(1+(u-4)^4\right)du = 13.671$

   +1 $\int_0^2 \ln\left(1+(u-4)^4\right)du$, +1 handles initial condition, +1 answer

   c. Write an equation for the line tangent to the curve at P.

   At $t = 2, m = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\ln 17}{12} = 0.326; y - 13.671 = 0.236(x-3)$ +1 slope, +1 equation

   d. For what value of t, if any, is the object at rest? Explain your reasoning.

   $x'(t) = 0$ if t = 0,4; $y'(t) = 0$ if t = 4; t = 4 +1 reason, +1 answer

2. A water tank at Camp Newton holds 1200 gallons of water at time t = 0. During the time interval $0 \leq t \leq 18$ hours, water is pumped into the tank at the rate $W(t) = 95\sqrt{t}\sin^2\frac{t}{6}$ gallons per hour. During the same time interval, water is removed from the tank at the rate $R(t) = 275\sin^2\frac{t}{3}$ gallons per hour.

   a. Is the amount of water in the tank increasing at time t = 15? Why or why not?

   No, the amount of water is not increasing at t = 15 since W(15) – R(15) = -121.09 < 0 +1 answer with reason

   b. To the nearest whole number, how many gallons of water are in the tank at time t = 18?

   $1200 + \int_0^{18}\left(W(t) - R(t)\right)dt = 1309.788 \rightarrow 1310$ gallons +1 limits, +1 integrand, +1 answer

   c. At what time t, for $0 \leq t \leq 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

   $W(t) - R(t) = 0 \rightarrow t = 0, 6.4948, 12.9748 \rightarrow (0,1200), (6.495, 525), (12.975, 1697), (18, 1310)$

   The values at endpts and critical pts show that abs min occurs at t = 6.494 or 6.495.

   +1 interior critical pts, +1 amount of water least at t = 6.494, +1 analysis for abs min

   d. For t > 18, no water is pumped into the tank, but water continues to be removed at the rate R(t) until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

   $\int_{18}^{k} R(t)dt = 1310$ +1 limits, +1 equation

3. The Taylor series about x = 0 for a certain function f converges to f(x) for all x in the interval of convergence. The nth derivative of f at x = 0 is given by $f^{n}(0)=\frac{(-1)^{n+1}(n+1)!}{5^{n}(n-1)^{2}}$ for $n \geq 2$. The graph of f has a horizontal tangent line at x = 0, and f(0) = 6.

a. Determine whether f has a relative maximum, a relative minimum, or neither at x = 0. Justify your answer.

f has a rel max at x = 0 because f ' (0) = 0 and f " (0) < 0. +1 answer, +1 reason

b. Write the third degree Taylor polynomial for f about x = 0.

$$f(0)=6 \rightarrow f'(0)=0 \rightarrow f''(0)=\frac{-3!}{5^{2}1^{2}}=\frac{-6}{25} \rightarrow f'''(0)=\frac{4!}{5^{3}2^{2}}$$

$$P(x)=6-\frac{3!x^{2}}{5^{2}2!}+\frac{4!x^{3}}{5^{3}2^{2}3!}=6-\frac{3}{25}x^{2}+\frac{1}{125}x^{3}$$

+3 P(x), <-1> each incorrect term, <-1> max for use of extra terms

c. Find the radius of convergence of the Taylor series for f about x = 0. Show the work that leads to your answer.

$$u_{n}=\frac{f^{n}(0)}{n!}x^{n}=\frac{(-1)^{n+1}(n+1)}{5^{n}(n-1)^{2}}x^{n}$$

$$\left|\frac{u_{n+1}}{u_{n}}\right|=\left|\frac{\frac{(-1)^{n+2}(n+2)}{5^{n+1}(n)^{2}}x^{n+1}}{\frac{(-1)^{n+1}(n+1)}{5^{n}(n-1)^{2}}x^{n}}\right|=\left(\frac{n+2}{n+1}\right)\left(\frac{n-1}{n}\right)^{2}\frac{1}{5}|x|$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_{n}}\right|=\frac{1}{5}|x|<1 \rightarrow |x|<5$$

The radius of convergence is 5.

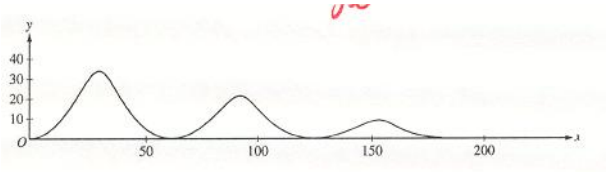
+1 general term, +1 sets up ratio, +1 computes limit, +1 applies ratio test to get radius of convergence

POW DUE 4/23/10

The use of a calculator is REQUIRED for these problems.

1. A particle moving along a curve in the plane has position (x(t), y(t)) at time t, where $\frac{dx}{dt} = \sqrt{t^4 + 9}$ and $\frac{dy}{dt} = 2e^t + 5e^{-t}$ for all real values of t. At time t = 0, the particle is at the point (4, 1).

a. Find the speed of the particle and its acceleration vector at time t = 0.
b. Find an equation of the line tangent to the path of the particle at time t = 0.
c. Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.
d. Find the x-coordinate of the position of the particle at time t = 3.



2. The figure below shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by $x(t) = 10t + 4\sin t$, $y(t) = (20 - t)(1 - \cos t)$, where x and y are measured in meters. The derivatives of these functions are given by $x'(t) = 10 + 4\cos t$, $y'(t) = (20 - t)\sin t + \cos t - 1$.

a. Find the slope of the path at time t = 2. Show the computations that lead to your answer.
b. Find the acceleration vector of the car at the time when the car's horizontal position is x = 140.
c. Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.
d. For $0 < t < 18$, there are two times at which the car is at ground level (y = 0). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

3. An object moving along a curve in the xy-plane has position (x(t), y(t)) at time t with $\frac{dx}{dt} = \cos(t^3)$ and $\frac{dy}{dt} = 3\sin(t^2)$ for $0 \le t \le 3$. At time t = 2, the object is at position (4, 5).

a. Write an equation for the line tangent to the curve at (4, 5).
b. Find the speed of the object at time t= 2.
c. Find the total distance traveled by the object over the time interval $0 \le t \le 1$.
d. Find the position of the object at time t = 3.